Dedicated to the memory of Prof. Mihail Renert

Design of the Elastic System of the Vibrating Screens

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This work presents the vibrating screens casing elastic supports requirements, the usual elastic materials (cylindrical coil springs, elastic rubber elements, airsprings) as well as some design approaches related to the bearings made of these kind of elements. The computational example shows the application method of the presented relations.

Keywords: vibrating screen, elastic bearing

The vibrating screens are frequently used in different fields of industrial activity (chemical, food, coal, metallurgic or construction materials industry) due to their performances (high specific flows for high efficiency and accuracy of the screening).

Besides the classical screening devices [1-4], many studies regarding new types of specialized vibrating screens are published. These latter devices are related to special and difficult materials (humid, fiber-like or fine dispersed materials) as well as to high efficiency of the screening. Rolling motion screens [5], oscillating screens [6], inversed spherical pendulum screens [7] are to be mentioned in this category. There are also studies regarding the behaviour of the material on the screen [8-10] or the efficiency and the accuracy of the screening process [3].

In order to build high performance screening devices a correlation between the functional parameters of the vibrations generator and the parameters of the casing elastic bearing is required.

In order to allow the free vibration of the screen's casing and in order to avoid the transmission of the vibrations to the supporting elements of the system, the fixed frame of the screening device, by means of some elastic materials, supports the casing of the screen. The elastic bearing system is done using coil springs, various spring types systems, [3, 11], elastic rubber elements [11-13], air springs [13], different combinations of them.

The elastic bearing of the vibrating screens is subjected to hard working conditions: high number of cycles fatigue (greater than 10^7, which is the usual design basis) [3], resonance (when stopping or starting the machine), dusty environment etc.

The elastic bearing requires a careful design and construction in order to fully reach its functional role:

- ensuring the elasticity and eigen pulsation required by the vibration regime parameters of the imposed screening process.

- ensuring the mechanical and fatigue strength which leads to the flawless behaviour of the machine throughout its entire lifespan.

This work presents several approaches regarding the design of the elastic bearing of the screening devices casing, considering the particular cases of screening device's vibrating regime, using either coil springs or elastic rubber elements.

Design requirements of the casing elastic bearing

The are several required elements involved in the design of the bearing system: eigen pulsations and vibration amplitudes of the oscillating system (steady working regime as well as resonance regime), choice of elastic element types and their system layout, strength calculus of the elastic elements, means of attaching the elements to the casing and the fixed frame.

Good vibration insulation is reached when the elastic elements ensure a low eigen pulsation. In the same time, the elastic elements should allow casing vibration amplitudes of 8…10 times larger than in the case of steady working regime (in order to ensure the safe passing of the resonance domain) [3]. Vibrations dampers should be considered for resonance regime.

When designing the elastic bearing one should take into account that the eigen vibration modes cannot get coupled. This leads to a significant calculus simplification. In order to reach the de-coupling of the eigen modes one should consider that (fig.1) [13]:

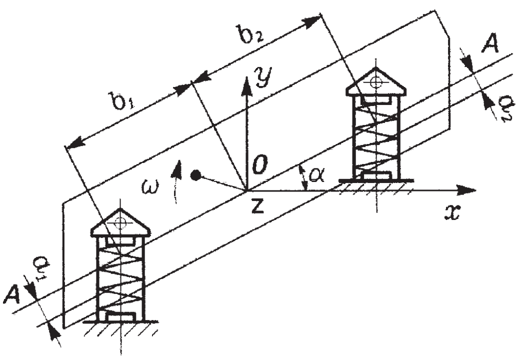

Fig. 1 Elastic bearings layout

- the elastic elements are positioned in symmetrical planes with respect to xOy and yOz planes (where O is the gravity center of the screen's casing). This means that the distances from O to the intersection points of the elastic elements' axis *A-A* (which passes through the gravity center) are equal to the inertia radius of the casing ρ_0 (on the plane xOy) $b_1 = b_2 = \rho_0$;

- the distances from the line *A-A* to the centers of the elastic elements should be zero $a_1 = a_2 = 0$;

- the elastic constants of the supporting elements on the directions Oy and Ox should be equal, $k_y = k_x$;

- the elastic constant k_y is chosen so that the eigen frequency of the screening device is 2.0…2.5 Hz (eigen pulsation p = 12.56…15.7 s^{-1}) [3]. In this way the fast passing of the resonance domain as well as a low transmissibility of the vibrations is reached.

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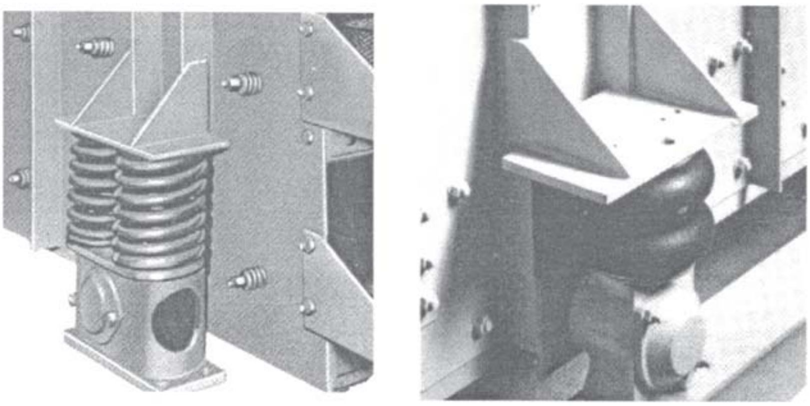

Fig. 2. Elastic bearing elements [14]:
a – compression coil springs; b – air springs

Elastic elements used for supporting the casing

Most frequently used elastic elements are: cylindrical coil springs, (fig.2.a, fig.3), air springs (fig.2.b), elastic rubber elements (fig. 6, fig. 7).

Cylindrical coil springs

The cylindrical coil sprigs are the most commonly used elastic bearing elements.

This type of bearing exhibits though some disadvantages such as: short life span, dynamic forces transmission to the foundation especially when at resonance. These disadvantages can be eliminated using the air springs.

Attaching the springs to the casing can be done as shown in figure 3.

Air springs

These air springs are made of adequate quality rubber, reinforced with high strength synthetic fibers. The inside air pressure is around 0,4…0,5 MPa.

A lot of advantages can be seen in the case of air spring supports: long life span, easy adjustable stiffness (just by adjusting the inside air pressure), reduction of the dynamic forces transmitted to the foundation, low noise emission, easy replacing of the elements in case of failure. [13] .

Oppositely to the coil springs, the air springs have a linear elastic characteristic. Increasing the vibration amplitude when passing through the resonance domain will increase the stiffness as well as the eigen frequency. This will lead to the reduction of the transient regime.

The bearing capacity can be adjusted by adjusting the inside air pressure. High bearing capacity means low stiffness. This feature ensures an almost unlimited domain of application of the air springs. For the vibrating screens, the most commonly used types of air springs with textile reinforcement are presented in figure 4. Type 1 is identical with type 1, with the only difference that the lower side is not working.

The graphical representation of the dependence between the elastic constant k_y and the force P_c which acts on the spring, for types *1*, *2*, and *3* from fig. 4, is shown in figure 5 [13].

Type 1 air springs can be used for a high range of loads, while the air pressure is relatively low.

The air pressure is chosen so that the screen stability on the horizontal plane and a minimal eigen frequency are ensured. The optimal air pressure is determined using relations [13]:

$$p_a = 50P_c + 29400 \quad Pa\text{, for type } 1;$$
$$p_a = 25P_c + 68600 \quad Pa\text{, for type } 2;$$
$$p_a = 50P_c + 4900 \quad Pa\text{, for type } 3.$$

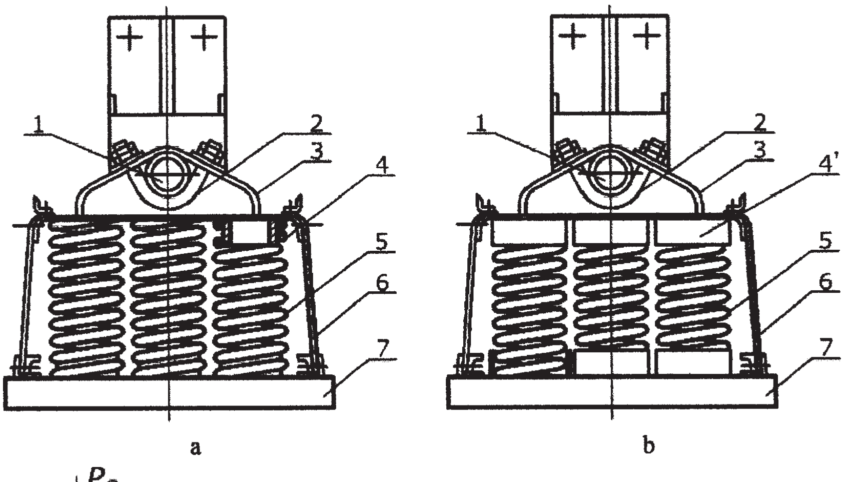

Fig. 3. Ways of attaching the springs to the screen and its fixed frame. [13]
a – with cylindrical wedges b – with cylindrical supports
1 – casing's bearing element, 2 – attaching element, 3 – bearing plate, 4 – cylindrical wedges, 4' – cylindrical supports, 5 – coil springs, 6 – rubber casing, 7 – fixed frame

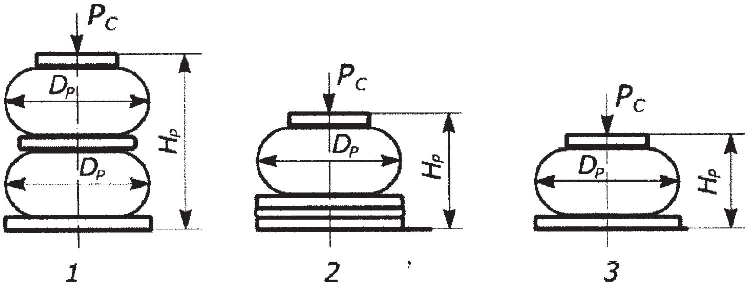

Fig. 4. Constructive types of air springs used for the elastic bearings of the screen's casings

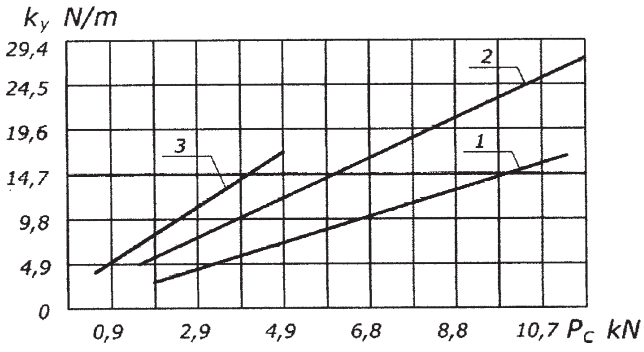

Fig. 5. Graphical representation of the dependence between the elastic constant k_y and the force P_c, for the air springs in fig. 4 [13]

Elastic rubber elements

These kind of elements, when compared to the coil springs, exhibit some advantages: nonlinear elastic characteristic, high energy dissipation capacity, low noise emissions.

Between the foundation and the bearing plate of the casing support cylindrical elastic rubber elements are places. Their symmetry axis is horizontal. In order to ensure their fixed position, different profiled elements are used. The elastic elements can move on horizontal direction, rotating inside the profiled elements. This has as an immediate effect the reduction of the horizontal dynamic forces transmitted to the foundation.

The largest displacement can be recorded during the transient regime, when the vibration amplitude suddenly increases. The central orifice increases the elasticity of the elements and ensures a better ventilation and cooling process (large area of thermal transfer).

Different types of elastic elements can be used as cylindrical elastic supports, and massive elastic supports.

Using these elements instead of coil springs, leads to a reduction of the resonance amplitude of 300% (from 45…50mm, in the case of coil springs, to 16mm), a reduction to 2…3 s of the resonance domain and an important lowering of the noise emissions [13].

The specific mechanical characteristics as well as the design of different elastic bearings that use rubber elements are presented in a more detailed manner [12].

The elastic bearing design

Theoretical elements

The inertial vibrating screens are monomassic dynamic systems excited by harmonic perturbation forces produced by vibrations generators with eccentrically disposed rotating masses.

The vibrations generator placed in the mass center of the system, produces a harmonic perturbation force, having the following modulus:

$$F_0 = M_0 \cdot R \cdot \omega^2 \qquad (1)$$

where:

M_0 - is the value of the eccentrically placed mass, or in the case of multiple masses, the sum of them;

R – Mass eccentricity (the distance between the rotation center and the mass centre of the eccentrically paced mass);

ω - perturbation force pulsation (the angular velocity of the vibrations generator shaft).

Being excited by the vibrations generator, the screen exhibits harmonic vibrations on the horizontal x and vertical y directions. The amplitudes are $A_{x,y}$ and the pulsation ω, is equal to the one of the perturbation force.

In the case of post-resonance regime (when the system damping can be neglected), the vibrations amplitudes are given by the ratio between the pulsation of the perturbation force and the eigen pulsation $k_{\omega/p} = \omega / p_{x,y} = 3...10$ [3, 15]:

$$A_{x,y} = \frac{M_0 \cdot R}{M + M_0} \cdot \frac{\omega^2}{\left| p_{x,y}^2 - \omega^2 \right|} \qquad (2)$$

where M is the vibrating mass of the screen and the eigen pulsations are [3, 11]

$$p_{x,y} = \sqrt{k_{x,y} / (M + M_0)} \qquad (3)$$

where $k_{x,y}$ are the bearing system's elastic constants on the horizontal and vertical direction.

The design of the elastic bearing system with compressed coil springs

The elastic bearing is subjected to vertical as well as to horizontal loading.

The eigen pulsation of the system on the vertical direction can be determined using the ratio $k_{\omega/p} = \omega / p_{x,y} = 3...10$, knowing its value and the value of the perturbation force pulsation. The elastic constant of the springs results from the elastic system eigen pulsation relation [3,11, 15]:

$$k_y = p_y^2 (M + M_0). \qquad (4)$$

The system eigen pulsation shouldn't be higher than $p_y = 12…22\ s^{-1}$ so that a low vibrations transmissibility to the foundations is reached [3, 15] .

The cylindrical coil spring elastic constant is [3, 11]

$$k_{0y} = \frac{G \cdot d^4}{8 \cdot D^3 \cdot i}, \qquad (5)$$

where

G – is transverse elasticity modulus of the spring material;

d – wire diameter;

D – winding mean diameter;

i – number of active windings.

The elastic constant of the spring system is:

$$k_y = u \cdot k_{0y} = \frac{G \cdot d^2 \cdot u}{8 \cdot D^3 \cdot i} \qquad (6)$$

where u – is the number of springs (an even number of springs should be chosen).

From relation (6), one can obtain the dimensioning relation of the spring's wire [3, 11]:

$$d = \frac{8 \cdot i \cdot c^3 \cdot k_y}{u \cdot G}, \qquad (7)$$

where, $c=D/d$ is the spring coefficient.

The experimental data obtained from field as well as the ones resulted form the fatigue strength studies, lead to values of c=6…10 [3, 11].

In order to set up the spring dimensions, which depend on the winding angle and the number of active windings (considered as a known value in relation (7)), the determination of the transverse elastic constant of the spring is required. This quantity depends on the kinematics of the machine's motion which is imposed by the technological factors.

Considering the fact that, inside the post-resonance domain, the damping effect in the elastic system is insignificant, for the eigen pulsation on the horizontal direction, one can use the following relation [3, 11] :

$$p_x = \omega \cdot \sqrt{1 - \frac{R}{A_x} \cdot \frac{M_0}{M + M_0}}. \quad (8)$$

Knowing the value of the eigen pulsation p_x, the transverse elastic constant of the spring system can be determined:

$$k_x = p_x^2 (M + M_0). \quad (9)$$

In the case of post-resonance regime with low damping, in order to obtain circular vibrations of the casing, the elastic constants of the system should be equal. In this case the value of k_x is equal to k_y given by relation (6).

The transverse elastic constant of a coil spring is [11]

$$k_{0x} = \frac{3 \cdot E \cdot I}{H_p^3 \cdot \chi}, \quad (10)$$

where,

$$H_p = \pi \cdot D \cdot i \cdot \sin\alpha; \quad (11)$$

$$\chi = \frac{2 + \mu \cdot \cos^2\alpha}{2\sin\alpha} \quad (12)$$

where:

α - is the winding angle of the axially pre-stressed spring;

I – cross-sectional inertia moment of the coil's transverse section, with respect to the axis perpendicular to the spring axis (for the circular cross-section $I = \pi \cdot d^4 / 64$);

E – longitudinal elasticity modulus of the spring material;

μ - Poisson's coefficient.

The transverse constant of the spring system, composed of *u* identical springs (*u* even number) is:

$$k_x = u \cdot k_{0x} = \frac{3 \cdot u \cdot E \cdot d}{32 \cdot \pi^2 \cdot c^3 \cdot i^3} \cdot \frac{1}{(2 + \mu) \cdot \sin^2\alpha - \sin^4\alpha}. \quad (13)$$

Using the notation:

$$B = \frac{32}{3} \cdot \frac{\pi^2 \cdot c^3 \cdot i^3}{u \cdot E \cdot d}, \quad (14)$$

from (13) one obtains (only the positive real solution is physically compatible):

$$\sin\alpha = \sqrt{\left[2 + \mu - \sqrt{(2 + \mu)^2 - 4/(B \cdot k_x)}\right] / (2 \cdot \mu)}. \quad (15)$$

The condition sin α < *1* leads to a minimal number of active windings (using (14) too):

$$i > \frac{1}{c} \sqrt{\frac{3 \cdot u \cdot E \cdot d}{64 \cdot \pi^2 \cdot k_x}}. \quad (16)$$

The winding angle of the non-tensioned spring is:

$$\alpha' = \alpha_0 + \alpha \quad (17)$$

where, the angle α is determined by relation (15) and the angle α_0, corresponding to the static displacement due to self weight of the vibrating part of the device is given by:

$$\sin\alpha_0 = \frac{1}{\pi \cdot D \cdot i \cdot u} \cdot \frac{(M + M_0) \cdot g}{k_y}. \quad (18)$$

The geometrical elements of the spring are:

- height of blocked spring:

$$H_b = (i_t - 0{,}5) \cdot d \quad (19)$$

where i_t is the total number of windings;

- height of free spring:

$$H_0 = H_b + i \cdot (h - d) \quad (20)$$

where *h* is the winding step;

The winding step is chosen to that during work the spring doesn't get blocked [13].

$$h = \frac{d + (1{,}15 \ldots 1{,}30) A}{i} \quad (21)$$

where *A* is the oscillations amplitude for steady working regime.

Considering that the resonance amplitude A_r is about 5…10 times larger than the steady regime amplitude *A*, (the resonance amplitude is obtained by introducing in the expression of the amplification factor the value ω / p = 1), when the non-blocking condition is imposed to the resonance regime too, in relation (21) one replaces *A* cu A_r.(especially at the machine stops, when the passing through resonance takes longer than in the case of machine start).

The static displacement of the spring (under the mobile equipment self-weight) is:

$$f_{st} = \frac{M + M_0}{k_y} = \frac{M + M_0}{k_{0y} \cdot u} \quad (22)$$

The spring height under the action of the static load is:

$$H_{st} = H_0 - f_{st} \quad (23)$$

It's required that $H_{st} - H_b > A$ (for resonance $H_{st} - H_b > A_r$).

The dynamic displacement varies between:

$$f_{d\,max} = f_{st} + A; \; f_{d\,min} = f_{st} - A, \quad (24)$$

where *A* is the vibrations amplitude of the mobile equipment.

The static load of one spring is:

$$F_{st} = \frac{M_e \cdot g}{u} + k_{0y} \cdot A \quad (25)$$

where:

M_e is the mass of the mobile equipment (considering the screened material too);

u – number of springs.

The shear stress in the spring wire for the normal working regime is [13]

$$\tau = \frac{8 \cdot F_{st} \cdot D}{\pi \cdot d^3}. \quad (26)$$

The strength condition should be met $\tau < \tau_{all}$ wehre τ_{all} = 150…180 MPa.

The maximal and minimal dynamic forces are determined as it follows:

$$F_{max} = k_{0y} \cdot f_{d\,max}, \quad F_{min} = k_{0y} \cdot f_{d\,min}. \quad (27)$$

The fatigue check of the spring is done using the already known methods.

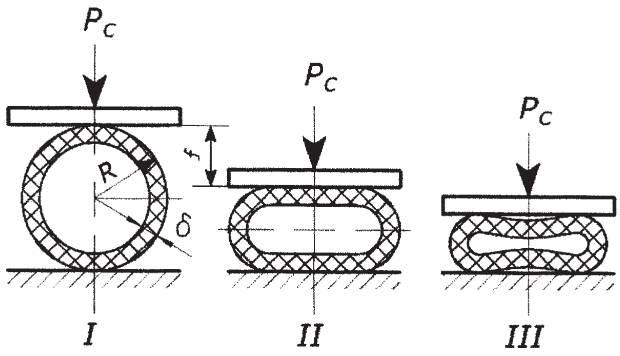

Fig. 6. Deformed shapes of the elastic element

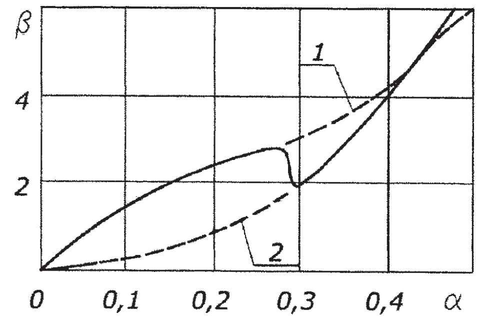

Fig. 7. α and β dependence [13]

As a conclusion, the design of the springs involves the following steps:

- choosing the material, the spring coefficient, the number of windings, the number of active windinds and the number of springs. Using relation (7) one determines the wire diameter;
- determining the winding angle using (17) and then the initially number of active windings is checked by relation (16);
- the strength and fatigue check is done (in the case of under or over-dimensioning of the spring, the spring coefficient will be changed and the design starts over again).

Elastic rubber elements design

One considers a rubber element compressed by two metallic plates (fig. 6). The compression parameters of the elastic element are defined by relations [13]:

$$\alpha = \frac{f}{R} \ ; \qquad (28)$$

$$\beta = \frac{12 P_c R^2}{E \cdot \delta^3} \cdot \qquad (29)$$

where:

f - is the displacement of the elastic element under the action of the static load Pc;

R – exterior radius of the elastic element;

δ - wall thickness (fig. 6).

The dependence between α and β is graphically represented in figure 7.

The deformed shape *II* from figure 6 is characterized by the curve *1* from figure 9. The deformed shape *III*, is characterized by curve *2*.

Knowing the static load P_c and the constructive parameters of the elastic element, using relation (31) one determines the parameter β. Using its value and the figure 7, the parameter α can be determined. Next, using (30) the displacement *f* corresponding to the static load P_c can be determined. The elasticity of the element results after that.

$$k_y = P_c / f \ . \qquad (30)$$

For $\alpha = \frac{f}{R} = 0{,}285$ the deformed shape, *II* becomes *III* and the dependence function β = f(α) is given by the continuous line in figure 7.

Computational example

One considers an inertial vibrating screen in the post-resonance regime, having the vibrating mass of 650 kg (the screened material weight is neglected).

The characteristics of the screened material and the technological conditions, impose circular vibrations of the screening device mobile equipment, having the amplitude $A_x = A_y = A = 2.5$ mm and the frequency $\omega = 100.55\ s^{-1}$ (n=960 rot/min). A vibration generator with the counterweight M_o = 14 kg is used.

The eigen pulsation of the system is:

$p_y = \omega/k_{\omega/p} = 100.55 / 5 = 20.11\ s^{-1}$where, for elimination of the transmission of the dynamic loadings to fundament of the screen, $k_{\omega/p} = \omega/p_y = 5$ [15].

The elastic constant of the springs system, on the vertical direction (relation 4) is $k_y = 2.7 \cdot 10^5$ N/m.

For the springs system one chooses: spring coefficient *c* = 7, number of active windings *i* = 4, number of springs *u* = 8, material is steel with $G = 8.1 \cdot 10^{10}\ N/m^2$.

The spring wire diameter will be (relation (7)): d = 0.005 m = 5 mm.

The value of *B* (relation (16)) is: $B = 1.73 \cdot 10^4$, where one considered E = 2.1 . 1011 N/m².

If the vibrations trajectory is circular, $k_x = k_{xy} = 2.7 \cdot 10^5$ N/m.

The value of the winding angle for the tensioned spring will be (relation (17)): α = 10.2°.

The spring's winding angle, corresponding to the static displacement due to self weight of the vibration part (relation (20))

D = c . d = 7 . 5 = 35 mm = 0.035 m), will be: $\alpha_o = 0.4°$.

The winding angle of the non-tensioned spring is α' = α+ α_o = 10.6°.

The minimal number of active windings condition (relation (18)) is fulfilled: i = 4 > 1.75.

Conclusions

The technological parameters of the screening process (flow, efficiency, screening accuracy) can be reached only if the dynamic parameters of the screen are well established (the amplitude, the frequency and the trajectory of the vibrations). In this case a very important role is given to the elastic bearing system. It has a great influence on the dynamic parameters through its elastic constant.

The elastic constant is chosen so that the eigen pulsation of the system ensures besides the dynamic and functional parameters, the fast passing through the resonance domain (starting and stopping of the machine) and the reduced transmission of dynamic forces to the foundation.

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